Antifragile Urban Mobility System: Implementation Report (System Performance Equilibrium - Model A)

September 2, 2025

1 Abstract

This report presents an auditable equilibrium model for antifragile urban mobility systems, derived from grounded theory analysis of 123 studies (2019–2024). The model quantifies how disruptions trigger adaptive gains, achieving an Antifragility Factor (AF) > 1.1 (10% post-disruption improvement). It combines a core dynamical equation with operationalized indicators, calibration, sensitivity analysis, and back-testing, ensuring EU compliance for Horizon Europe deliverables. Key contributions: traceable KPIs, a reproducibility pack, and a transport equilibrium annex.

2 Indicator Table & Data Dictionary

All indicators are normalised on a rolling 30-day window:

$$\hat{x} = \frac{x - x_{\min}}{x_{\max} - x_{\min}}$$

Unless noted, variables within $\bar{E}(x)$ are their normalised forms; hats are omitted for readability.

Variable	Scale	Indicator Definition	Data Source	Unit	Aggregation Window
M(t)	Individual / Network	Mobility throughput: avg. multi-modal trips per capita per hour	Smart-card logs; CDR; GPS probes	trips h^{-1} cap^{-1}	Daily average
R(t)	Network	Redundancy ratio: dormant links / active links (pre-event)	OpenStreetMap; traffic sensors	–	Weekly snapshot
A(t)	System	Adaptation velocity: ∂(modal-share diversity)/∂t	Household surveys	% · day^{-1}	Bi-weekly
C(t)	System	Network entropy of OD matrix	CDR; Bluetooth sensors	nats	Daily
D_{eff}(t)	Event	Disturbance (domain/recency/source-weighted)	D2.1 event catalogue + operator/official logs	–	Hourly → daily aggregate
S(t)	System	Stress: Σ(vol/cap)·speed_drop	Loop detectors	–	Hourly
Q(t)	Equity	Stratified Theil–T of accessibility minutes	Census; GTFS	–	Monthly
P(t)	Demand-management	Composite index in [0,1] from time-varying fares via min–max by mode/time, aggregated with ridership weights.	Operator APIs, open fares	index (0–1)	Daily index
E_energy(t)	Energy	EV-grid resilience score	Grid SCADA	–	Daily
E_ICT(t)	ICT	Telecom network uptime during event	Operator logs	%	Event-specific
I(t)	Cross-layer	Synergy index: $\chi \cdot \sum_{\ell \neq m} \text{Distance}(\ell, m)^{-1}$	Multilayer graph	–	Weekly

3 Parameter JSON (excerpt for submission; full in code archive)

```
{
  "alpha1": 0.5, "alpha2": 0.5, "alpha3": 0.5, "alpha4": 0.5,
  "beta1": 0.4, "beta2": 0.3,
  "omega1": 0.2, "omega2": 0.2,
  "zeta": 0.1, "theta": 0.1, "pi": 0.1,
  "sigma": 0.5, "chi": 1.0,
  "kappa": 0.1,
  "delta_transport": 1.0, "delta_environment_weather": 0.9, "
     delta_utilities": 0.9, "delta_social": 0.9,
  "tau_days": 6,
  "omega_src_official": 1.0, "omega_src_operator": 0.9, "
     omega_src_sensor": 0.9, "omega_src_social": 0.6, "
     omega_src_survey": 0.6
}
```

Notation: ω_1, ω_2 are infrastructure weights (energy, ICT). ω_{src} refers to source reliability weights in the disturbance mapping. These are unrelated.

4 Core Equation

The antifragile potential $E(t)$ is an aggregate KPI $\bar{E}(x)$ per period, evolving via relaxation:

$$\dot{E} = -\kappa \left(E - \bar{E}(x)\right)$$

where the target is:

$$\begin{aligned}\bar{E}(x) = \; & \alpha_1 M(t) + \alpha_2 R(t) + \alpha_3 A(t) + \alpha_4 \, \psi(C(t)) \\ & - \beta_1 D_{\text{eff}}(t) - \beta_2 S(t) \\ & + \omega_1 E_{\text{energy}}(t) + \omega_2 E_{\text{ICT}}(t) \\ & + \zeta I(t) - \theta Q(t) + \pi P(t)\end{aligned}$$

$C(t)$ denotes the entropy of the origin–destination (OD) flow distribution, measuring how evenly trips are spread across OD pairs (each OD pair = one ordered origin–destination cell in the OD matrix).

$$\psi(C) = 1 + \sigma \, H_{\text{norm}}, \qquad H_{\text{norm}} = \frac{C}{\log(N_{\text{OD}})},$$

where C is Shannon entropy (C in nats), N_{OD} is the number of OD pairs, and σ is a scaling factor. This ensures $H_{\text{norm}} \in [0, 1]$.
With $\kappa = 0.1$ (as documented in the parameter table; typical convergence in ~ 10 periods), the discrete update rule is:

$$E_{t+1} = E_t - \kappa \left(E_t - \bar{E}(x_t)\right).$$

Example. Consider a network with 10 zones. The number of OD pairs is $N_{\text{OD}} = 10 \times 9 = 90$. Suppose the Shannon entropy of the observed OD distribution is $C = 3.5$ nats. Then

$$H_{\text{norm}} = \frac{3.5}{\log(90)} \approx 0.75, \qquad \psi(C) = 1 + 0.5 \times 0.75 = 1.375$$

for $\sigma = 0.5$. Assume the baseline performance is $E_0 = 0.60$, the recovery target is $\bar{E} = 0.67$ (raised, in part, due to the high diversity reflected in $\psi(C) = 1.375$), and $\kappa = 0.1$.
The first update step is:

$$E_1 = E_0 - \kappa \left(E_0 - \bar{E}\right) = 0.60 - 0.1\,(0.60 - 0.67) = 0.60 - 0.1 \times (-0.07) = 0.607.$$

Thus the system moves 10% of the way from the current state (0.60) toward the target (0.67). Repeated iterations converge monotonically to $\bar{E}$, as $|1 - \kappa| = 0.9 < 1$ ensures stability.

5 Calibration Protocol

• Two-stage: (1) AHP expert elicitation (e.g., 20 scholars / 10 practitioners) for relative weights. (2) Elastic-net regression on 2019–2024 events to maximise out-of-sample R^2 on ΔE.
• Final weights as a convex combination: 0.6 expert + 0.4 data (document ranges and cross-validated λ).
• Implementation: Python; dynamics via a discrete update; archive code and seeds.

6 Sensitivity Analysis (Sobol)

Global sensitivity on mean E and settling time (time to $|E - \bar{E}| \leq 5\%$ of the initial gap). First-order S_i measures individual impact; total-order S_{T_i} includes interactions.
Example placeholder results (replace with project-run outputs):

Metric	**Top Parameters**	**Values**
First-Order (mean E)	K_R, R, α_4, E_{ICT}, α_3	0.1260, 0.1223, 0.0759, 0.0371, 0.0307
Total-Order (mean E)	K_R, R, α_4, E_{ICT}, α_3	0.1614, 0.1573, 0.1035, 0.0564, 0.0482

Table 1: Worked User Equilibrium (UE) solution for a two-link network with demand $Q = 100$, used as illustrative annex material (cf. Wardrop, 1952; Beckmann, McGuire & Winsten, 1956).

Include a tornado plot of sensitivity coefficients; archive code and random seeds for reproducibility.

UE Annex (Transport Equilibrium)

User Equilibrium (UE) solution with demand $Q = 100$ and link costs

$$c_1 = 10 + 0.1 f_1, \qquad c_2 = 12 + 0.04 f_2.$$

At equilibrium,

$$10 + 0.1 f_1 = 12 + 0.04(100 - f_1) \;\Rightarrow\; 0.14 f_1 = 6 \;\Rightarrow\; f_1 \approx 42.857.$$

Thus

$$f_1 \approx 42.857, \qquad f_2 = 100 - f_1 \approx 57.143,$$

with common cost

$$c \approx 14.286.$$

(Wardrop, 1952; Beckmann, McGuire & Winsten, 1956)

7 Back-Testing (Template)

Calibrate on City A / Event Type 1; validate on City B / Event Type 2. Report prediction for AF and error vs. observed AF. Include a small ablation (e.g., drop R or C) to show contribution to AF. (Replace this template with the project's actual cases.)

8 Risks & Limitations

• Data sparsity: coverage <70% degrades accuracy; apply imputation and report uncertainty.
• Policy inertia: demand-management (P) changes may lag events; document policy timelines.
• External validity: calibrated primarily on mid-size European cities; caution on transfer.
• Model scope: extreme 'black swan' cascades are not explicitly modelled.

9 Reproducibility & Compliance Appendix

(i) GDPR Note: perform a DPIA; apply k-anonymity (k=5) and, where appropriate, differential privacy ($\varepsilon \approx 0.1$); no raw personal data shared.
(ii) Data Management Summary: follow FAIR principles; provide anonymised sample datasets via EU-compliant portals.
(iii) Code Archive: Python script for dynamics and sensitivity (NumPy/Scipy); sample dataset; README with run instructions (e.g., run main.py with params.json). Archive DOI to be added.

10 Implementation Pseudocode

For each time step t:

1. Read inputs
$$x(t) = [M, R, A, C, D_{\text{eff}}, S, Q, P, E_{\text{energy}}, E_{\text{ICT}}, I].$$

2. Compute the target
$$\begin{aligned} \bar{E} &= \alpha_1 M + \alpha_2 R + \alpha_3 A + \alpha_4\, \psi(C) - \beta_1 D_{\text{eff}} - \beta_2 S \\ &\quad + \omega_1 E_{\text{energy}} + \omega_2 E_{\text{ICT}} + \zeta I - \theta Q + \pi P, \\ \psi(C) &= 1 + \sigma \cdot H_{\text{norm}}, \qquad H_{\text{norm}} = \tfrac{H}{H_{\max}}. \end{aligned}$$

3. Update performance
$$E \;\leftarrow\; E - \kappa\,(E - \bar{E}).$$

4. Log E, $\bar{E}$, and inputs for plots and checks.

11 Equilibrium Model Integrated with D2.1 Event Mapping - Partner Guide

This guide integrates Deliverable D2.1's event mapping directly into the equilibrium model. All definitions are plain-language with examples, and nothing in the original report is removed.

11.1 Plain-language overview (D2.1-native)

The model tracks a performance score E(t) for the urban mobility system. D2.1 produced a catalogue of events (domain, scale, severity, time, source, AOI). We convert those events into a composite disturbance D_eff(t) that, together with stress S(t) on affected areas, shifts the target score $\bar{E}(x)$. E(t) then moves gradually toward $\bar{E}(x)$ at speed κ, like a thermostat approaching a set-point.

11.2 Event→Model linkage (core)

For an event e with domain d, severity level $L \in \{1, \dots, 5\}$, start time t_e, and source type src, define

$$D_{\text{eff}}(e,t) = \omega_{\text{src}}(\text{src}) \cdot \exp\Big(-\tfrac{t-t_e}{\tau}\Big) \cdot \delta_d \cdot \frac{L-1}{4}.$$

Aggregate across concurrent events:

$$D_{\text{eff}}(t) = \sum_e D_{\text{eff}}(e,t).$$

Here $D_{\text{eff}}(t)$ replaces raw disturbance terms and co-varies with $S(t)$ computed on links within the event's area(s) of interest (AOIs).

11.3 Normalisation — step-by-step

Indicators are normalised over a rolling 30-day window using

$$\hat{x} = \frac{x - x_{\min}}{x_{\max} - x_{\min}} \in [0,1].$$

The baseline x_0 is tracked separately for antifragility (AF) comparisons.
Example. In the last 30 days, trips per capita per hour ranged 0.8–1.6. Today $x = 1.3$. Then

$$\hat{x} = \frac{1.3 - 0.8}{1.6 - 0.8} = \frac{0.5}{0.8} = 0.625.$$

Note: always use $x_{\min}$ from the 30-day window, not day-1 x_0 (which is tracked separately for AF).

11.4 Worked example (with D2.1 mapping)

Baseline: E(t)=0.60, κ=0.10. D2.1 detects a level-4 transport event 12h ago, source=operator; τ=6 days; δ_transport=1.0; ω_src=0.9.
Compute $\rho = \exp(-12/144)$=0.920; event contribution = 0.9×0.920×(4−1)/4 = 0.621.
Suppose resilience and policy inputs shift $\bar{E}(x)$ from 0.60 to 0.67.

Extending the horizon to $t = 11$ with $\kappa = 0.2$ gives

$$E_{11} \approx 0.661 \quad \Rightarrow \quad AF \approx 1.102 > 1.1.$$

11.5 Variable dictionary - what to send & examples

Code	**Plain-language name**	**What it means / how to compute**	**Unit**	**Frequency**	**Typical source**	**Simple example**
$M(t)$	Mobility throughput	Average multi-modal trips per person per hour (sum trips across modes ÷ population ÷ hours).	trips h^{-1} cap^{-1}	Daily average	Smart-card, CDR, GPS probes	2.4M trips/person/day with pop 1.0M → 2.4 trips/person/day ≈ 0.10 trips/person/hour.
$R(t)$	Redundancy ratio	Dormant links ÷ active links in pre-event snapshot (links that can carry flow but are idle vs. carrying).	–	Weekly	OSM topology + counts, sensors	300 active, 60 dormant → R = 0.20.
$A(t)$	Adaptation velocity	Rate of change of modal-share diversity over time; finite difference of a diversity index per day.	$\% \cdot \text{day}^{-1}$	Bi-weekly	Household/intercept surveys	Diversity 0.62 vs 0.58 over 14 days → (0.62−0.58)/14 ≈ 0.0029 day^{-1}.
$C(t)$	Network entropy of OD matrix	Shannon entropy of OD flows, normalised by maximum; higher = flows are more spread-out.	nats (normalised)	Daily	CDR, Bluetooth, APC/AVL	If flows are evenly distributed over 4 ODs, H is high; if 90% on one OD, H is low.
$D_{\text{eff}}(t)$	Event disturbance	Domain/recency/source-weighted disturbance index.	–	Hourly → daily aggregate	D2.1 event catalogue + operator/official logs	Transport event, $L = 4$, 12h old, operator source, $\tau = 6$d, $\delta = 1.0$, $\omega_{\text{src}} = 0.9$ →≈ 0.621.
$S(t)$	Stress index	Σ over links in AOIs of (vol/cap) × speed_drop; aggregates congestion and capacity strain.	–	Hourly → daily aggregate	Loop detectors, floating-car data	If vol/cap= 0.9 and speed_drop= 0.2 on a link → 0.18; sum across links.
$Q(t)$	Equity penalty	Stratified Theil-T of accessibility minutes across income or neighbourhood groups.	–	Monthly	Census + GTFS travel times	If low-income groups face 15% longer mean access times, Q increases.
$P(t)$	Demand-management	Composite index in [0,1] from time-varying fares via min–max by mode/time, aggregated with ridership weights.	index (0–1)	Daily	Operator APIs, open fares	Peak surcharge +20% → index rises (e.g., 0.65→0.72).
$E_{\text{energy}}(t)$	EV-grid resilience	Composite of uptime × spare capacity for EV charging during events.	–	Daily	Grid SCADA, DSO reports	Uptime 99.5% × capacity index 0.8 →≈ 0.796.
$E_{\text{ICT}}(t)$	ICT uptime (during event)	Percentage uptime of telecom/-control systems critical to operations.	%	Event-specific	Operator logs / incident reports	30 minutes downtime in a 10-hour event → 95% uptime.
$I(t)$	Cross-layer synergy	Graph-based proximity between modal layers (e.g., bike–rail hubs); sum of inverted transfer distances.	–	Weekly	Multilayer transport graph	If new hubs cut average transfer distance by 20%, I increases.

11.6 Parameter list — meanings and handling

Parameter	Meaning / role	Typical source / note
$\alpha_1 \ldots \alpha_4$	Weights on M, R, A, ψ(C)	Elicited via AHP; refined via elastic-net on 2019–2024 events.
$\beta_1,\ \beta_2$	Weights on D_eff and S (penalties)	Expect $\beta>0$; calibrated against outcomes.
$\omega_1,\ \omega_2$	Weights on energy and ICT resilience	Higher values increase target performance under reliable infra.
ζ	Weight on synergy I	Captures benefit of cross-modal integration.
θ	Weight on equity penalty Q	Higher θ penalises inequitable access.
π	Weight on demand management P	Reflects effectiveness of pricing/management levers.
σ	Strength of entropy boost ψ(C)=1+σ·H_norm	How much flow diversity improves resilience.
χ	Scaling inside synergy index I	Tune to keep I within [0,1] after normalisation.
κ	Adjustment speed of E toward $\bar{\text{E}}$	κ=0.1 implies ∼10 periods to close most of the gap.
δ (domain weights)	Relative impact of domains on D_eff	Init from D2.1 prevalence & impact; refine with elastic-net.
τ (recency half-life)	Decay of older events: exp($-\Delta$t/τ)	Choose so past week dominates; cross-validate.
ω_src (source reliability)	Weight by source: official/operator > sensor > social/survey	Set tiers during pilot; combine with ESS-based shrinkage.

11.7 Short glossary

- Antifragility Factor (AF): post-event E divided by pre-event baseline; AF>1 means improvement (≥10% often flagged as strong).
- User Equilibrium (UE): all used routes have equal travel time; unused routes would be slower.
- Entropy of flows: how spread out trips are across OD pairs (higher = more diverse).
- AOI (Area of Interest): the spatial footprint an event affects (e.g., 1 km^2 grid cells).

11.8 Data packaging checklist

We provide the following for each city/partner:
1) Indicator CSVs with: date_time, value, unit, source, coverage_pct, notes.
2) Event log CSV (from D2.1) with: event_id, domain, scale, severity_level (1–5), source_type, start_time, aoi_grid_id, ess_weight, notes.

3) Policy log CSV with: policy_id, start_time, end_time, policy_type (pricing/priority/etc.), affected_modes, parameters.
4) README with data sources, known gaps, preprocessing steps, and contact person.

11.9 Spatial & temporal resolutions (core)

Spatial: 1 km^2 grid aggregation for urban centres and mixed-use areas; AOIs snapped to grid and network links.
Temporal: hourly updates for transport-facing indicators; baselines computed on rolling 6-month windows.

11.10 Feedback requested

- Are variable definitions feasible with your data? If not, propose proxies.
- Are κ and the 30-day normalisation window appropriate?
- Which weights (α,β,δ,τ,ω_src,...) feel too strong/weak for your city?
- Should we add city-specific indicators (e.g., safety incidents) or AOI rules?

11.11 UE annex — plain-language

At UE, no traveller can shorten their trip by switching routes; used routes equalise in travel time. In this model, D_eff and S influence how quickly systems depart from/return to UE; ψ(C) and I increase when flows diversify across the network.

11.12 Quality & privacy notes

- Aim for $\geq$70% temporal/spatial coverage per indicator; annotate gaps.
- Apply k-anonymity (k$\geq$5) before sharing; no raw personal identifiers.
- Use ISO-8601 timestamps and consistent timezones.

12 Explanatory Notes: Antifragility Threshold and Model Linearity

AF threshold (why AF > 1.1?): We compute the Antifragility Factor as AF = E_post / E_pre. The project uses AF > 1.1 to flag a ≥10% post-disruption improvement. This pragmatic threshold aims to exceed typical noise/seasonality in E and reflect a material, policy-relevant gain; it can be tightened or relaxed during calibration if empirical variability warrants.
Is the model nonlinear? The relaxation step E_{t+1} = E_t − κ (E_t − Ē(x_t)) is linear in E, but the inputs x_t are nonlinear functions of data: D_eff uses exponential recency and weights; ψ(C) depends on normalised entropy; S aggregates link-level nonlinearities. Hence, the overall input–output mapping is nonlinear even though the convergence mechanism is linear (stable for 0<κ<2, ensuring convergence).

13 Worked Example: Using the Equilibrium to Achieve Antifragility

Goal: show a clear, numerical example of (i) the baseline, (ii) what a disruption does, and (iii) how targeted actions can make the system antifragile (post-disruption performance improves by >10%).

14 Assumptions (simple and explicit)

- **Performance score:** $E(t) \in [0, 1]$ (higher = better).
- **Update rule:**

$$E(t+1) = E(t) - \kappa\,[E(t) - \bar{E}(x_t)], \qquad \kappa = 0.20.$$

- **D2.1-native disturbance:**

$$D_{\text{eff}}(e,t) = \omega_{\text{src}} \cdot \exp\bigl(-\tfrac{\Delta t}{\tau}\bigr) \cdot \delta_{\text{domain}} \cdot \frac{L-1}{4},$$

  aggregated across events.

- **Parameters for this example:** $\tau = 6$ days; $\omega_{\text{src}} = 0.9$ (operator); $\delta_{\text{transport}} = 1.0$; severity $L = 4$ (High).
- **Penalty weights in $\bar{E}(x)$:** β_1 on D_{eff} and β_2 on stress S; use $\beta_1 = 0.08$, $\beta_2 = 0.10$.

15 Baseline (before the event)

Assume the system is steady: Ē = 0.60 and E(0) = 0.60.

16 Disruption and new target

Event at t=0: transport domain, severity L=4 starting 12 hours ago ⇒ $\rho = \exp(-12/144) \approx 0.920$. D_eff ≈ 0.9 × 0.920 × 0.75 = 0.621.
Given $\beta_1 = 0.08$, $\beta_2 = 0.10$, $D_{\text{eff}} \approx 0.621$, and $S = 0.10$:

$$\Delta\bar{E} = -\beta_1 D_{\text{eff}} - \beta_2 S = -\big(0.08 \times 0.621 + 0.10 \times 0.10\big) = -(0.04968 + 0.01000) \approx -0.060.$$

With baseline $\bar{E}_{\text{base}} = 0.60$,

$$\bar{E}_{\text{event}} = \bar{E}_{\text{base}} + \Delta\bar{E} = 0.60 - 0.060 \approx 0.54.$$

Immediate effect: E(1) = 0.60 − 0.20 × (0.60 − 0.54) = 0.588.

17 Antifragile response (raise the target above baseline)

Actions: detour routing and pop-up bus lanes (↑ redundancy R), coordinated bike-rail hubs (↑ synergy I), smart peak pricing (↑ P), targeted services (↓ Q).
Combined effect lifts the target to E_recovery = 0.67 (> 10% above baseline).

18 Recovery trajectory with $\kappa = 0.20$ and constant $\bar{E}_{\text{recovery}}$

Time (t)	Target Ē(t)	E(t)
0 (baseline)	0.60	0.60
1 (event)	0.54	0.588
2	0.67	0.604
3	0.67	0.618
4	0.67	0.628
5	0.67	0.636
6	0.67	0.643
7	0.67	0.649
8	0.67	0.653
9	0.67	0.656
10	0.67	0.659
11	0.67	0.661

By t=11, E(t) ≈ 0.661. Antifragility Factor AF = E_{post}/E_{pre} ≈ 0.661 / 0.60 = 1.102 (> 1.1).

19 Key takeaways

- The equilibrium step is simple: each period, E moves 20% toward the current target.
- Disturbance strength (D_{eff}) and stress (S) lower the target during the event.

• Structural improvements (R, I, P, lower Q) raise the target above the old baseline—this is the essence of antifragility.
• Maintaining successful measures keeps the new, higher target in place so E(t) converges above the pre-event level.

Explanatory note. The quantity $\bar{E}_{\text{recovery}}$ is the new post–event target level of system performance, reflecting structural improvements (e.g. higher redundancy R, stronger integration I, adaptive pricing P, lower demand Q). The observed state $E(t)$ is updated each period via $E_{t+1} = E_t - \kappa(E_t - \bar{E}(x_t))$, so with $\kappa = 0.20$ the system moves 20% closer to the target at each step. If the target remains fixed at $\bar{E}_{\text{recovery}} = 0.67$, then $E(t)$ converges monotonically toward this value. In equilibrium, $E = \bar{E}_{\text{recovery}}$, which lies above the old baseline (0.60), yielding an antifragility factor $AF = E_{\text{post}}/E_{\text{pre}} > 1.1$.